



Outline: Normalization

- Redundant information and update anomalies
- Function dependencies
- Normal forms
 - 1NF, 2NF, 3NF
 - BCNF (Boyce Codd normal form)
- Lossless join property

Reading:

14.1.2 Redundant ... update anomalies

14.2.1 Functional dependencies

14.2.2 Inference rules for FDs

14.2.3 Equivalence of sets of FDs

14.2.4 Minimal sets of FDs

14.3 Normal forms based on PKs

Motivation:

Certain relation schemas have redundancy and update anomalies

- they may be difficult to understand and maintain

Normalization theory recognizes this and gives us some principles to guide our designs

Normal Forms: 1NF, 2NF, 3NF, BCNF, ... are each an improvement on the previous ones in the list

Normalization is a process that generates higher normal forms. Denormalization moves from higher to lower forms and might be applied for performance reasons.

Suppose we have the following relation

EmployeeProject

<u>ssn</u>	<u>pnumber</u>	hours	ename	plocation
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This is similar to *Works_on*, but we have included *ename* and *plocation*

Suppose we have the following relation

EmployeeDepartment

ename	<u>ssn</u>	bdate	address	dnumber	dname
-------	------------	-------	---------	---------	-------

This is similar to *Employee*, but we have included *dname*

In the two prior cases with EmployeeDepartment and EmployeeProject, we have **redundant** information in the database ...

- if two employees work in the same department, then that department name is replicated
- if more than one employee works on a project then the project location is replicated
- if an employee works on more than one project his/her name is replicated

Redundant data leads to

- additional space requirements
- update **anomalies**

Suppose EmployeeDepartment is the only relation where department name is recorded

insert **anomalies**

- adding a new department is complicated unless there is also an employee for that department

deletion **anomalies**

- if we delete all employees for some department, what should happen to the department information?

modification **anomalies**

- if we change the name of a department, then we must change it in all tuples referring to that department

If we design a database with a relation such as EmployeeDepartment then we will have complex update rules to enforce.

- difficult to code correctly
- will not be as efficient as possible

Such designs mix concepts.

For example, EmployeeDepartment mixes the Employee and Department concept

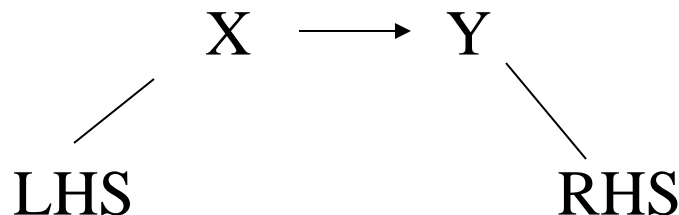
Section 14.2 Functional dependencies

Suppose we have a relation R comprising attributes X, Y, ...

We say a functional dependency exists between the attributes X and Y,

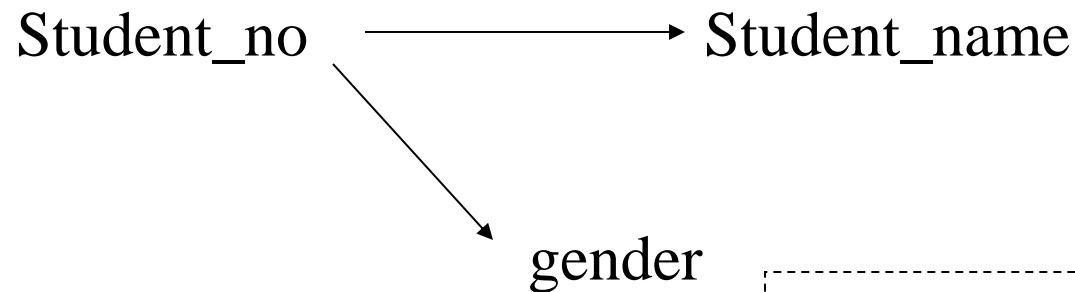
$$X \longrightarrow Y$$

if, whenever a tuple exists with the value x for X, it will always have the same value y for Y.



Student

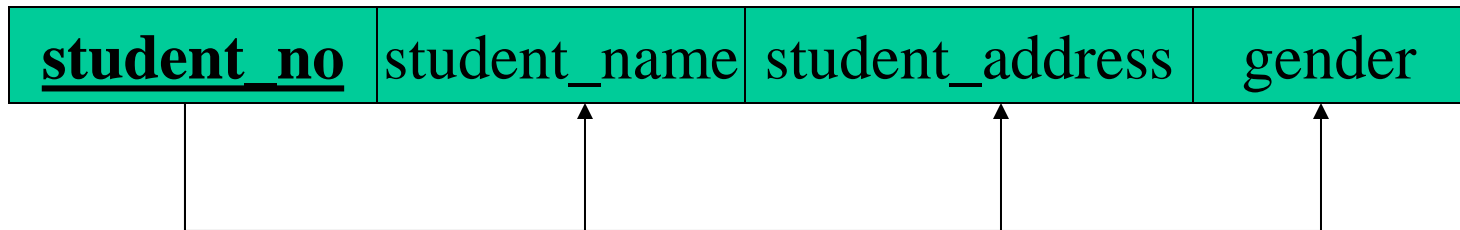
<u>course_no</u>	<u>student_no</u>	student_name	gender
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Given a specific student number, there is only one value for student name and only one value for gender found with it.

We always have functional dependencies between any candidate key and the other attributes.

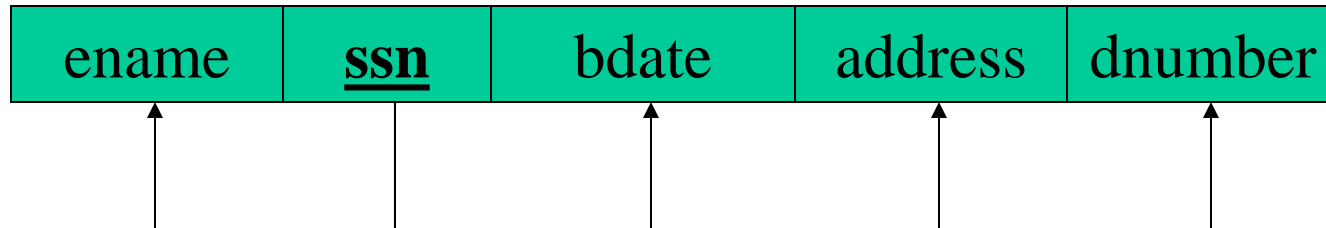
Student



student_no is unique ... given a specific *student_no* there is only one *student name*, only one *student address*, only one *gender*

Student_no \rightarrow student_name,
Student_no \rightarrow student_address,
Student_no \rightarrow gender

Employee

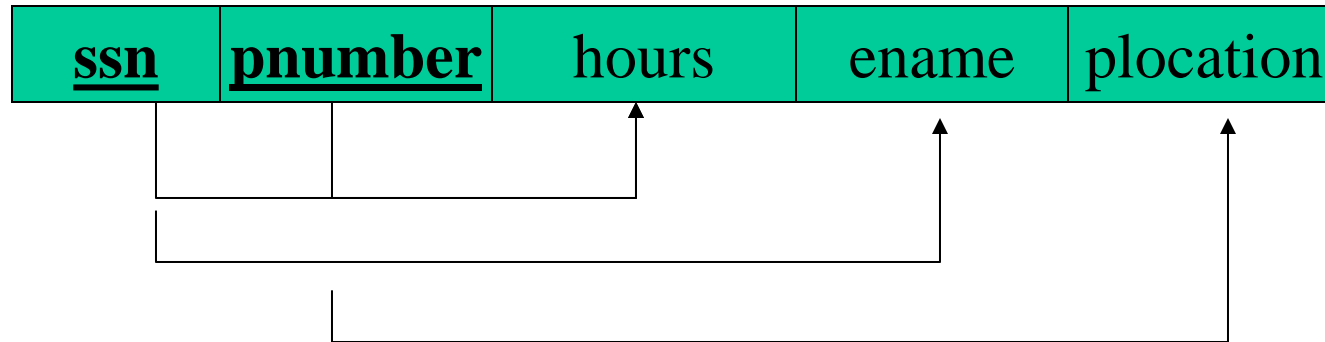


ssn is unique ... given a specific *ssn* there is only one *ename*, only one *bdate*, etc

$ssn \rightarrow ename,$
 $ssn \rightarrow bdate,$
 $ssn \rightarrow address,$
 $ssn \rightarrow dnumber.$

Suppose we have the following relation

EmployeeProject



This is similar to *Works_on*, but we have included *ename*, and we know that *ename* is functionally dependent on *ssn*.

We have included *plocation* ... functionally dependent on *pnumber*

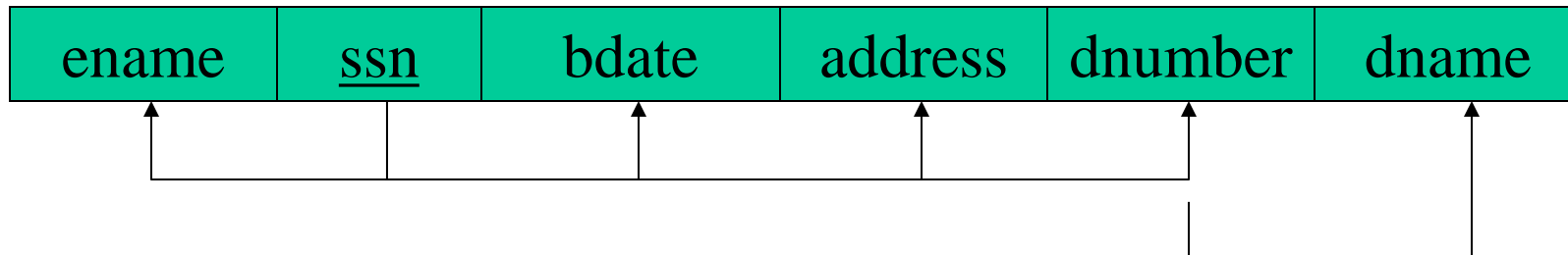
$\{ssn, pnumber\} \rightarrow hours,$

$ssn \rightarrow ename,$

$pnumber \rightarrow plocation.$

Suppose we have the following relation

EmployeeDept



This is similar to *Employee*, but we have included *dname*, and we know that *dname* is functionally dependent on *dnumber*, as well as being functionally dependent on *ssn*.

$ssn \rightarrow ename,$ $ssn \rightarrow bdate,$
 $ssn \rightarrow address,$ $ssn \rightarrow dnumber,$
 $dnumber \rightarrow dname.$ **$ssn \rightarrow dname$**

Minimal sets of FDs

- every dependency has a single attribute on the RHS
- the attributes on the LHS of a dependency are minimal
- we cannot remove a dependency without losing information.

Inference Rules for Function Dependencies

- From a set of FDs, we can derive some other FDs

Example:

$$F = \{ \text{ssn} \rightarrow \{ \text{Ename}, \text{Bdate}, \text{Address}, \text{dnumber} \}, \\ \text{dnumber} \rightarrow \{ \text{dname}, \text{dmgrssn} \} \}$$

 *inference*

$$\text{ssn} \rightarrow \text{dnumber}, \\ \text{dnumber} \rightarrow \text{dname}. \\ \text{ssn} \rightarrow \{ \text{dname}, \text{dmgrssn} \},$$

- F^+ (closure of F): The set of all FDs that can be deduced from F (with F together) is called the closure of F .

Inference Rules for Function Dependencies

- Inference rules:

- IR1 (reflexive rule): If $X \supseteq Y$, then $X \rightarrow Y$. ($X \rightarrow X$.)

- IR2 (augmentation rule): $\{X \rightarrow Y\} \models ZX \rightarrow ZY$.

- IR3 (transitive rule): $\{X \rightarrow Y, Y \rightarrow Z\} \models X \rightarrow Z$.

- IR4 (decomposition, or projective, rule):

$$\{X \rightarrow ZY\} \models X \rightarrow Y, X \rightarrow Z.$$

- IR5 (union, or additive, rule): $\{X \rightarrow Y, Y \rightarrow Z\} \models X \rightarrow ZY$.

- IR6 (pseudotransitive rule): $\{X \rightarrow Y, WY \rightarrow Z\} \models WX \rightarrow Z$.

Equivalence of Sets of FDs

E and F are equivalent if $E^+ = F^+$.

Minimal sets of FDs

- Every dependency has a single attribute on the RHS
- The attributes on the LHS of a dependency are minimal
- We cannot remove any dependency from F and still have a set of dependencies that is equivalent to F.

<u>ssn</u>	<u>pnumber</u>	hours	ename	plocation
------------	----------------	-------	-------	-----------

$\{ \text{ssn}, \text{pnumber} \} \rightarrow \text{hours},$
 $\text{ssn} \rightarrow \text{ename},$
 $\text{pnumber} \rightarrow \text{plocation}.$

Normal Forms

- A series of normal forms are known that have, successively, better update characteristics.
- We'll consider 1NF, 2NF, 3NF, and BCNF.
- A technique used to improve a relation is decomposition, where one relation is replaced by two or more relations. When we do so, we want to eliminate update anomalies without losing any information.

1NF - First Normal Form

The domain of an attribute must only contain atomic values.

- This disallows repeating values, sets of values, relations within relations, nested relations, ...
- In the example database we have a department located in possibly several locations: department 5 is located in Bellaire, Sugarland, and Houston.
- If we had the relation

Department

<u>dnumber</u>	dname	dmgrssn	dlocations
5	Research	333445555	Bellaire, Sugarland, Houston

then it would not be 1NF because there are multiple values to be kept in *dlocations*.

1NF - First Normal Form

If we have a non-1NF relation we can *decompose* it, or modify it appropriately, to generate 1NF relations.

There are 3 options:

- **option 1:** split off the problem attribute into a new relation (create a DepartmentLocation relation).

Department

<u>dnumber</u>	dname	dmgrssn
5	Research	333445555

DepartmentLocation

<u>dnumber</u>	<u>dlocation</u>
5	Bellaire
5	Sugarland
5	Houston

Generally considered the best solution

1NF - First Normal Form

- **option 2:** store just one value in the problem attribute, but create additional rows so that the other values can be stored too (department 5 would have 3 rows)

Department

<u>dnumber</u>	dname	dmgrssn	<u>dlocation</u>
5	Research	333445555	Bellaire
5	Research	333445555	Sugarland
5	Research	333445555	Houston

Dlocation becomes part of PK

Redundancy is introduced!
(not in 2NF)

1NF - First Normal Form

- **option 3:** if a maximum number of values is known, then create additional attributes so that the maximum number of values can be stored. (each location attribute would hold one location only)

Department

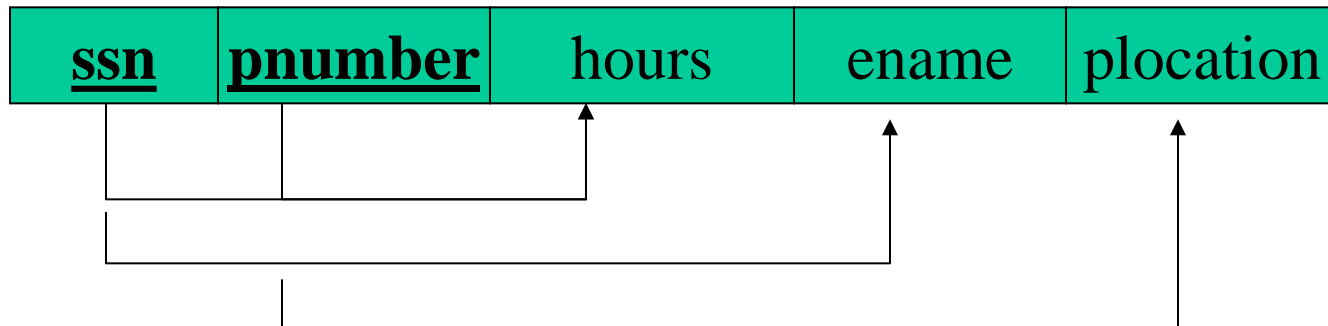
<u>dnumber</u>	dname	dmgrssn	dloc1	dloc2	dloc3
5	Research	333445555	Bellaire	Sugarland	Houston

2NF - Second Normal Form

- full functional dependency

$X \rightarrow Y$ is a full functional dependency if removal of any attribute A from X means that the dependency does not hold any more.

EmployeeProject



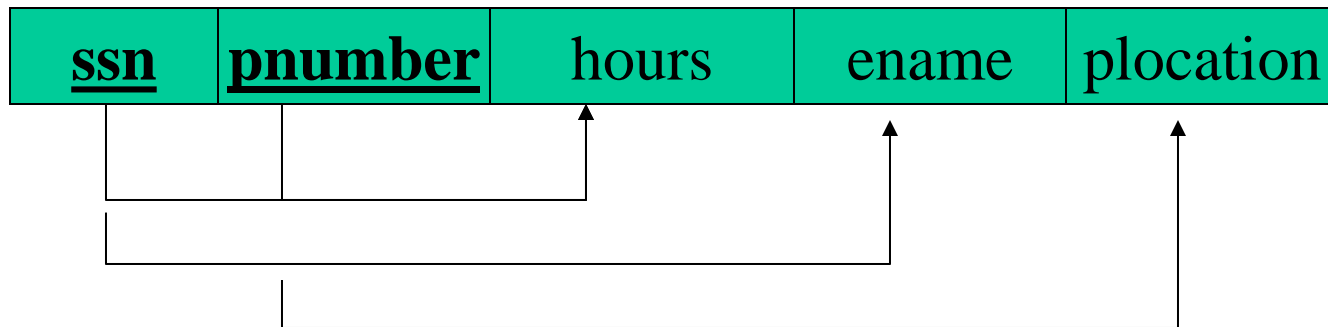
$\{ssn, pnumber\} \rightarrow hours$ is a full dependency
(neither $ssn \rightarrow hours$, nor $pnumber \rightarrow hours$).

2NF - Second Normal Form

- partial functional dependency

$X \rightarrow Y$ is a partial functional dependency if removal of some attribute A from X does not affect the dependency.

EmployeeProject



$\{ssn, pnumber\} \rightarrow ename$ is a partial dependency because $ssn \rightarrow ename$ holds.)

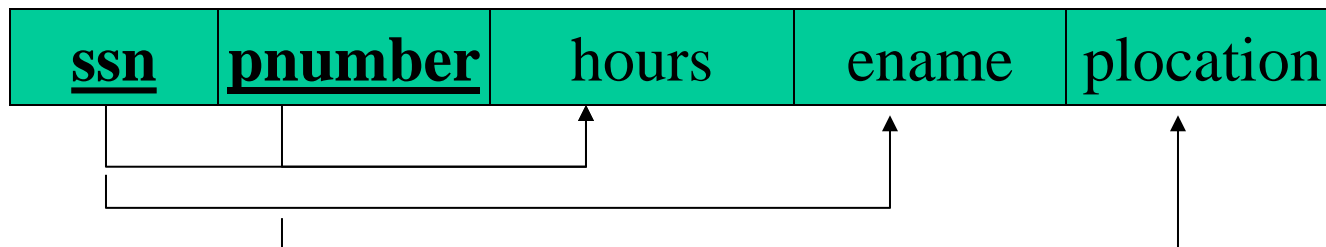
2NF - Second Normal Form

A relation schema is in 2NF if

- (1) it is in 1NF and
- (2) every non-key attribute must be fully functionally dependent on the candidate key.

If we had the relation

EmployeeProject



then this relation would not be 2NF because of two separate violations of the 2NF definition:

- *ename* is functionally dependent on *ssn*, and
- *plocation* is functionally dependent on *pnumber*



- *ename* is not fully functionally dependent on *ssn* and *pnumber* and
- *plocation* is not fully functionally dependent on *ssn* and *pnumber*.

{*ssn*, *pnumber*} is the primary key of EmployeeProject.

2NF - Second Normal Form

- We correct this by decomposing the relation into three relations - splitting off the offending attributes - splitting off partial dependencies on the key.

EmployeeProject



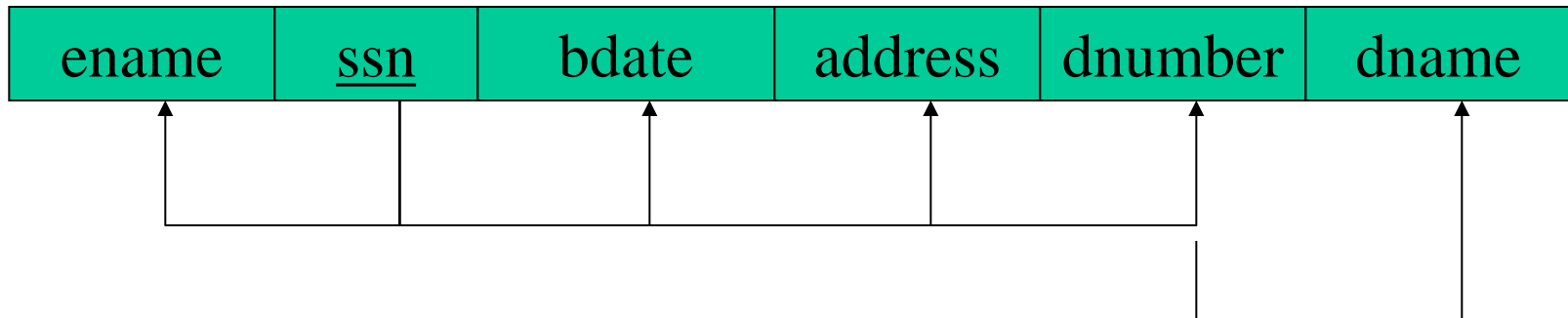
2NF

3NF - Third Normal Form

- Transitive dependency

A functional dependency $X \rightarrow Y$ in a relation schema R is a transitive dependency if there is a set of attributes Z that is not a subset of any candidate key of R , and both $X \rightarrow Z$ and $Z \rightarrow Y$ hold.

EmployeeDept



$ssn \rightarrow dnumber$ and $dnumber \rightarrow dname$

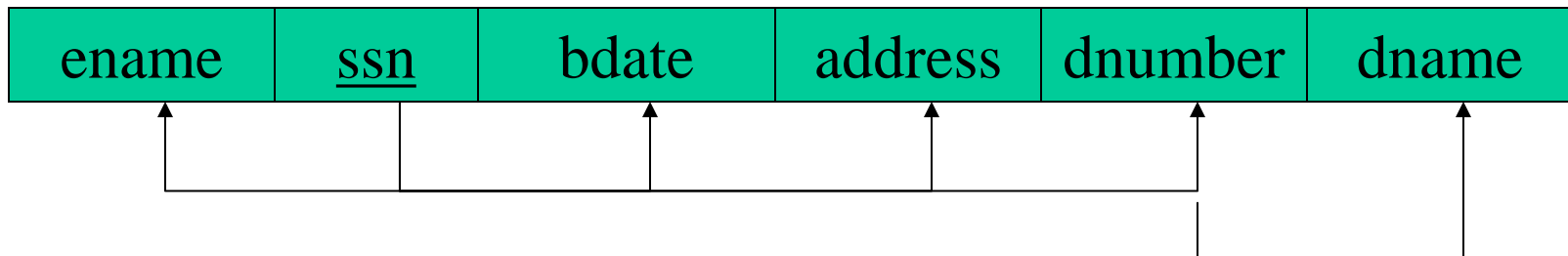
3NF - Third Normal Form

A relation schema is in 3NF if

(1) it is in 2NF and

(2) each non-key attribute must not be fully functionally dependent on another non-key attribute (there must be no transitive dependency of a non-key attribute on any candidate key.)

If we had the relation



then this relation would not be 3NF because

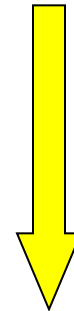
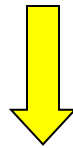
- *dname* is functionally dependent on *dnumber* and neither is
- a key attribute

3NF - Third Normal Form

- We correct this by decomposing - splitting off the transitive dependencies

EmployeeDept

ename	<u>ssn</u>	bdate	address	dnumber	dname
-------	------------	-------	---------	---------	-------



ename	<u>ssn</u>	bdate	address	dnumber
-------	------------	-------	---------	---------

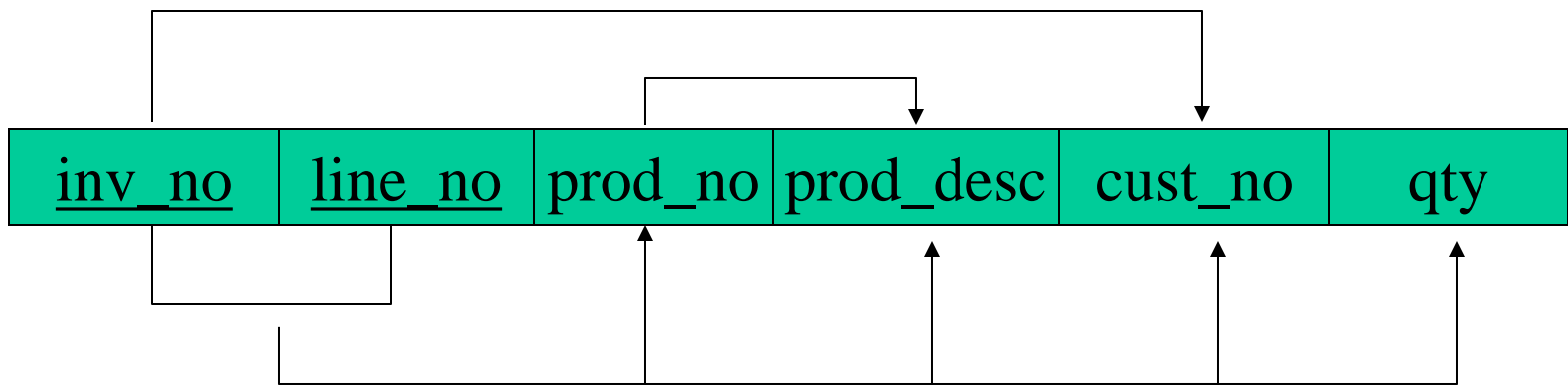
<u>dnumber</u>	dname
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3NF

Consider:

What normal form is it in?

What relations will decomposition result in?



$\{inv_no, line_no\} \rightarrow prod_no,$

inv_no: invoice number

$\{inv_no, line_no\} \rightarrow prod_desc,$

line_no: invoice line number

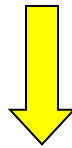
$\{inv_no, line_no\} \rightarrow cust_no,$

$\{inv_no, line_no\} \rightarrow qty,$

$inv_no \rightarrow cust_no, prod_no \rightarrow prod_desc$

Change it into 2NF:

<u>inv_no</u>	<u>line_no</u>	prod_no	prod_desc	cust_no	qty
---------------	----------------	---------	-----------	---------	-----

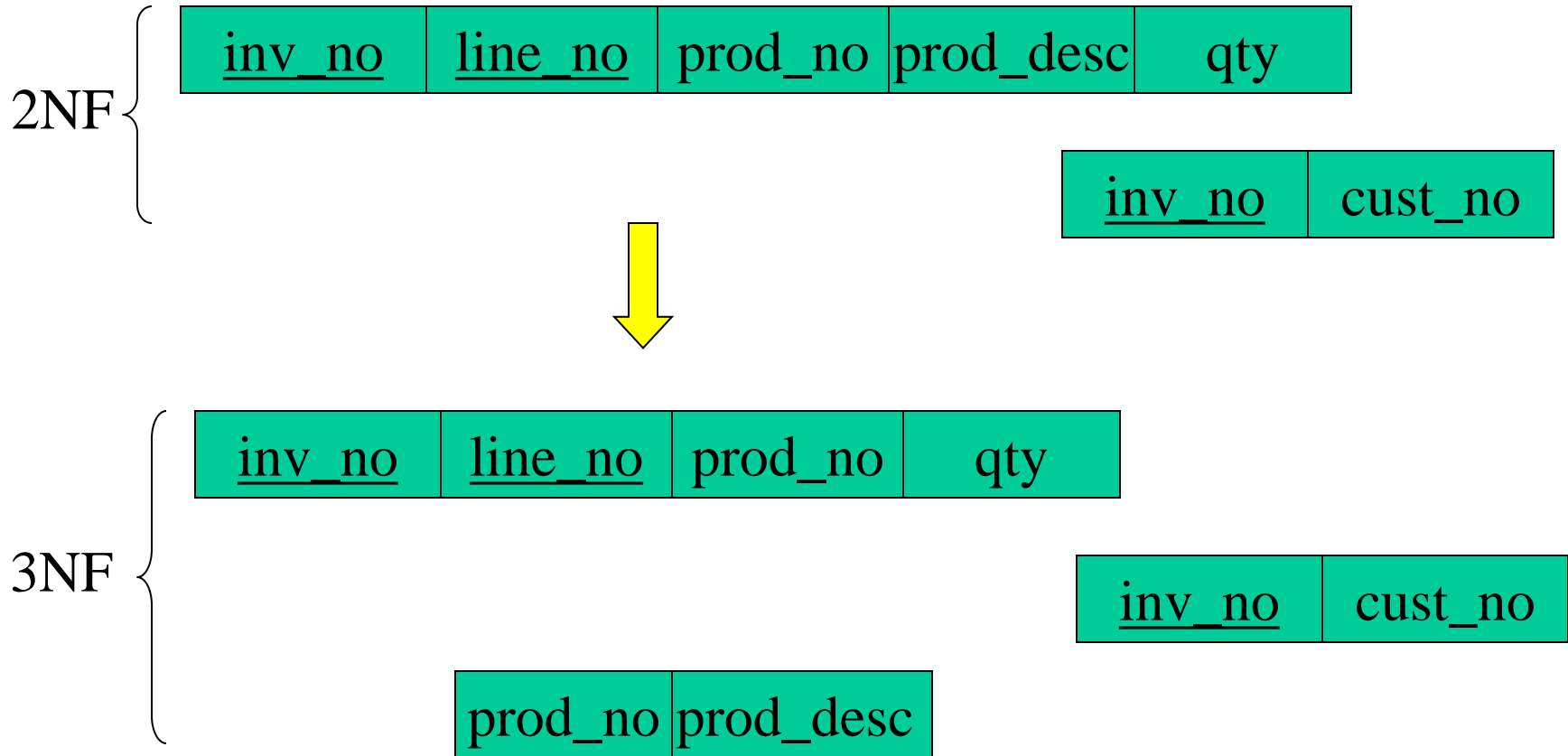


2NF {

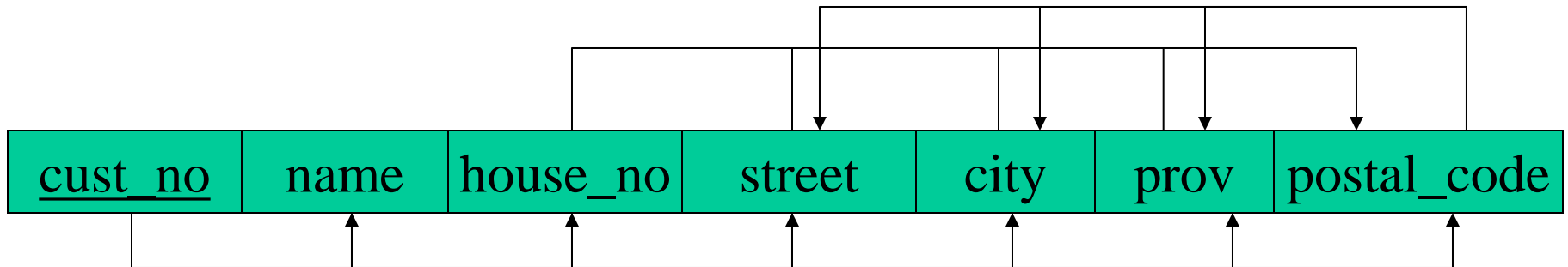
<u>inv_no</u>	<u>line_no</u>	prod_no	prod_desc	qty
---------------	----------------	---------	-----------	-----

<u>inv_no</u>	cust_no
---------------	---------

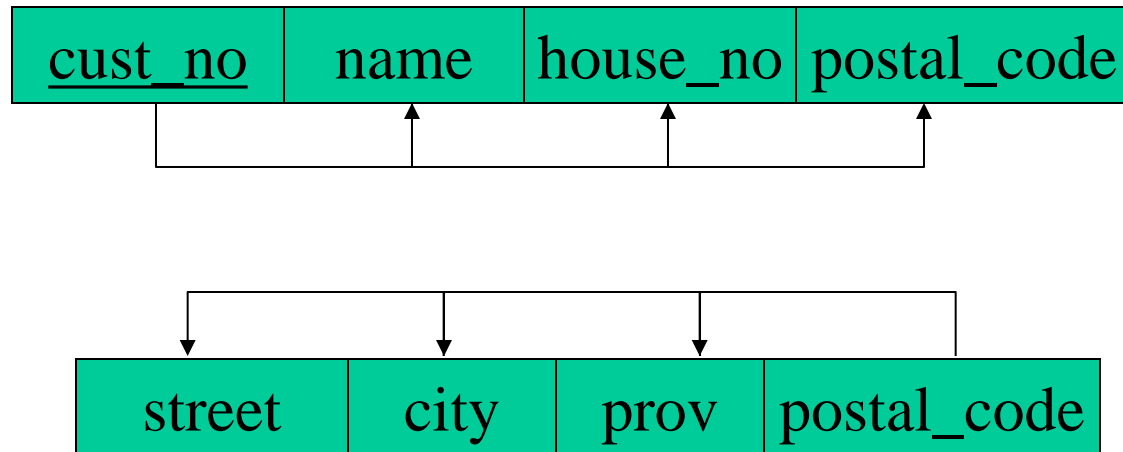
Change it into 3NF:



Consider:



Normalization



Boyce Codd Normal Form, BCNF

- Consider a different definition of 3NF, which is equivalent to the previous one.

A relation schema R is in 3NF if, whenever a function dependency $X \rightarrow A$ holds in R , either

- (a) X is a superkey of R , or
- (b) A is a prime attribute of R .

A superkey of a relation schema $R = \{A_1, A_2, \dots, A_n\}$ is a set of attributes $S \subseteq R$ with the property that no tuples t_1 and t_2 in any legal state r of R will have $t_1[S] = t_2[S]$.

An attribute is called a prime attribute if it is a member of any key.

Boyce Codd Normal Form, BCNF

- Consider a different definition of 3NF, which is equivalent to the previous one.

A relation schema R is in 3NF if, whenever a function dependency $X \rightarrow A$ holds in R , either

- (a) X is a superkey of R , or
- (b) A is a prime attribute of R .

There is no non-key attribute Y partially depends on a key X .
There is no non-key attribute Y transitively depends on a key X .

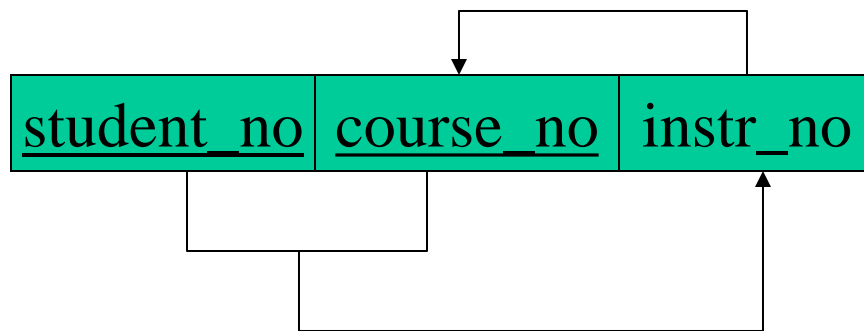
(A functional dependency $X \rightarrow Y$ in a relation schema R is a transitive dependency if there is a set of attributes Z that is not a subset of any key of R , and both $X \rightarrow Z$ and $Z \rightarrow Y$ hold.)

Boyce Codd Normal Form, BCNF

- If we remove (b) from the previous definition for 3NF, we have the definition for BCNF.
- A relation schema is in BCNF if every determinant is a superkey key. Stronger than 3NF:
 - no partial dependencies
 - no transitive dependencies where a non-key attribute is dependent on another non-key attribute
 - no non-key attributes appear in the LHS of a functional dependency.

Boyce Codd Normal Form, BCNF

Consider:



Instructor teaches one course only.

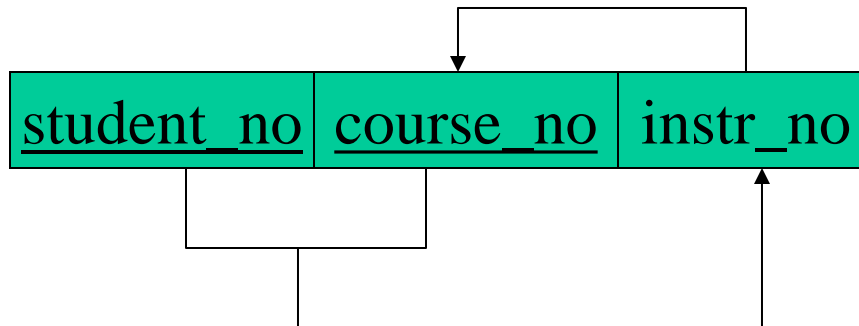
In 3NF!

Student takes a course and has one instructor.

$\{\text{student_no}, \text{course_no}\} \rightarrow \text{instr_no}$
 $\text{instr_no} \rightarrow \text{course_no}$

Boyce Codd Normal Form, BCNF

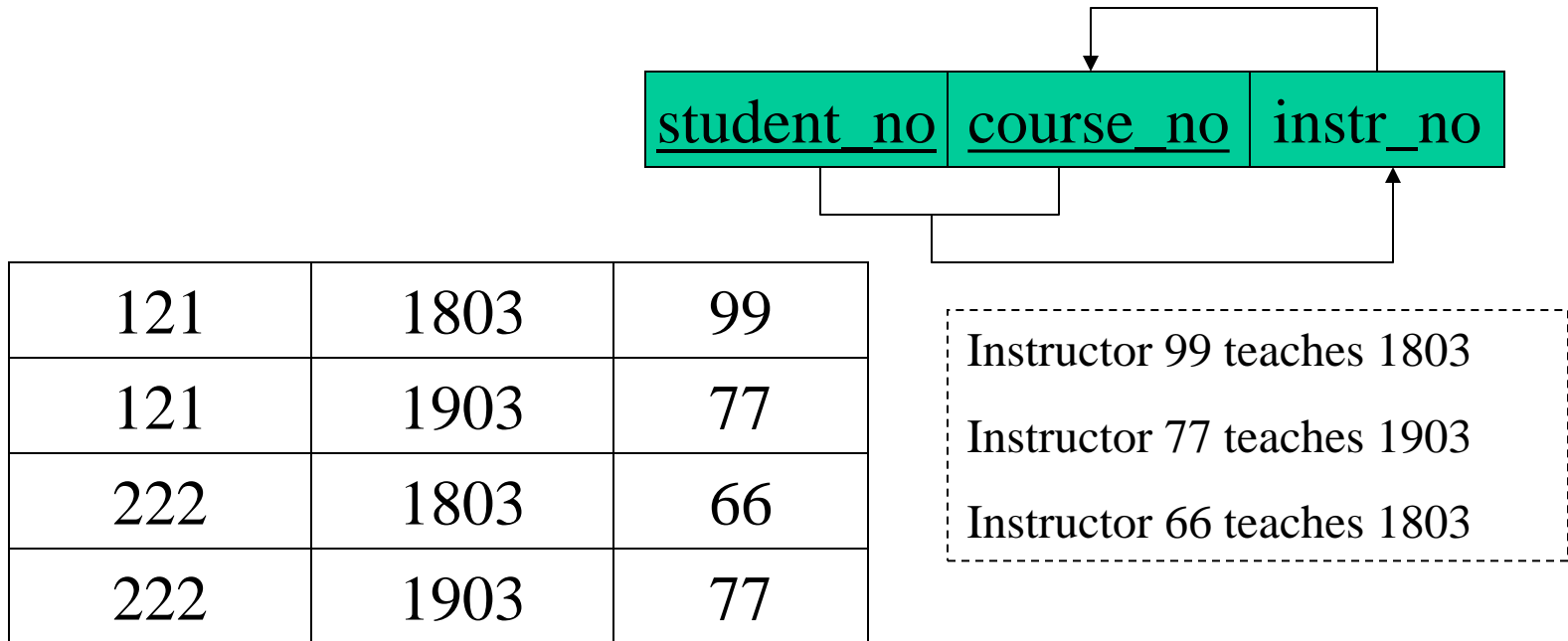
Some sample data:



121	1803	99
121	1903	77
222	1803	66
222	1903	77

Instructor 99 teaches 1803
Instructor 77 teaches 1903
Instructor 66 teaches 1803

Boyce Codd Normal Form, BCNF



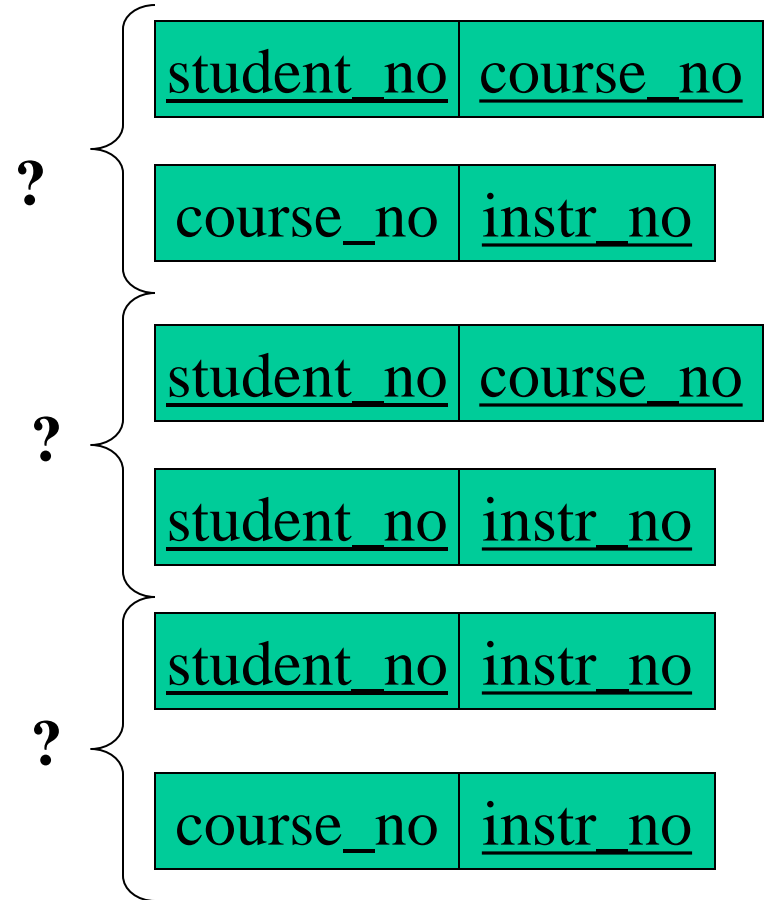
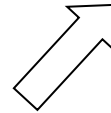
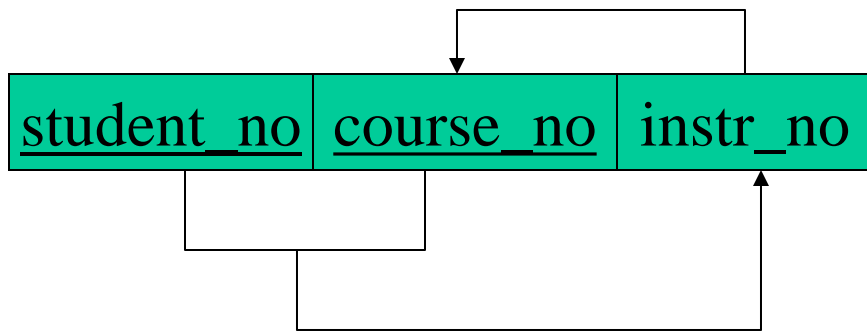
Deletion anomaly: If we delete all rows for course 1803 we'll lose the information that instructor 99 teaches student 121 and 66 teaches student 222.

Insertion anomaly: How do we add the fact that instructor 55 teaches course 2906?

Boyce Codd Normal Form, BCNF

How do we decompose this to remove the redundancies? - without losing information?

Note that these decompositions do lose one of the FDs.



Boyce Codd Normal Form, BCNF

Which decomposition preserves all the information?

S#	C#
121	1803
121	1903
222	1803
222	1903

C#	I#
1803	99
1903	77
1803	66

?

<u>student_no</u>	<u>course_no</u>
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course_no	<u>instr_no</u>
-----------	-----------------

<u>student_no</u>	<u>course_no</u>
-------------------	------------------

<u>student_no</u>	<u>instr_no</u>
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<u>student_no</u>	<u>instr_no</u>
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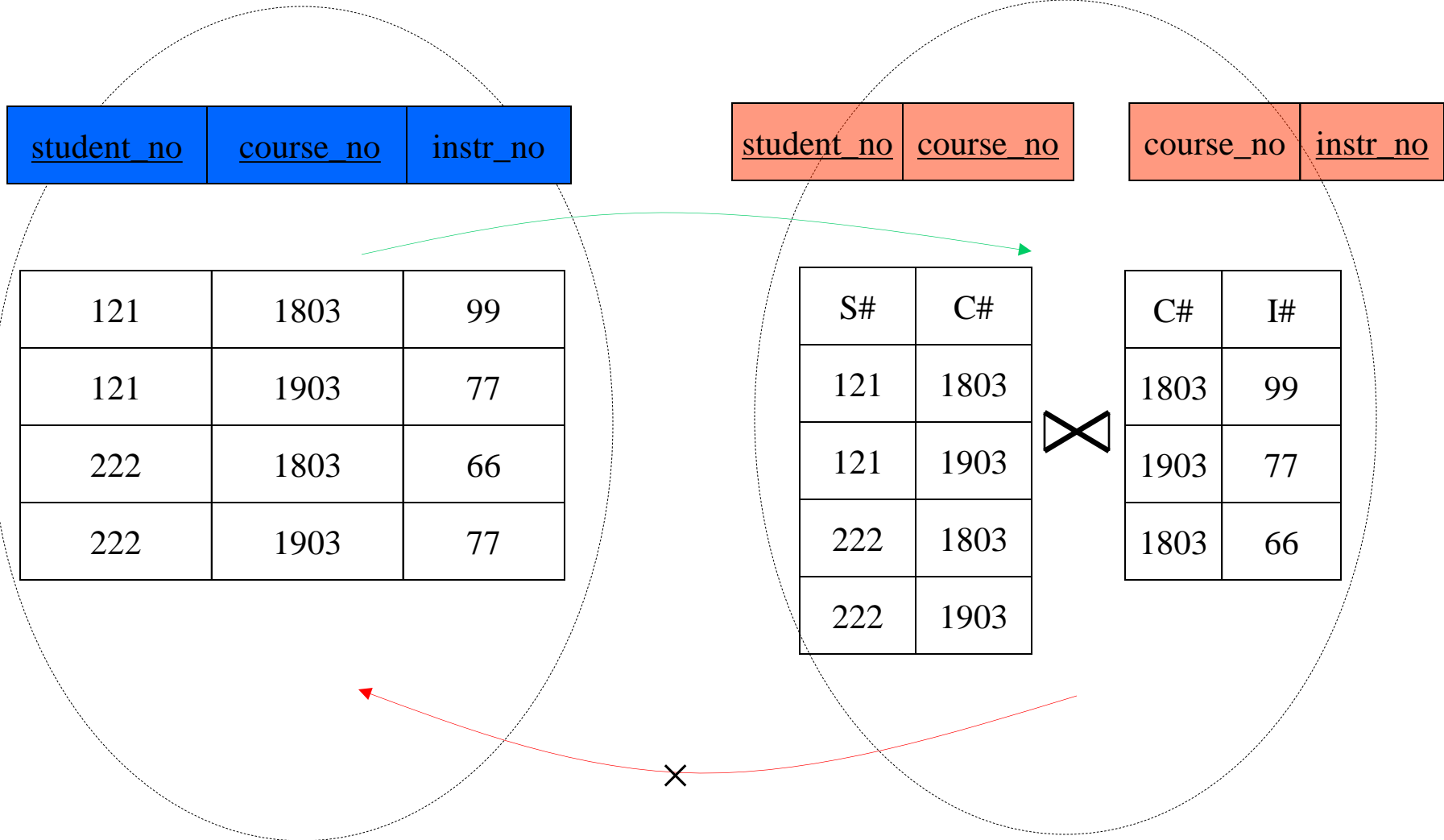
course_no	<u>instr_no</u>
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Joining these two tables leads to *spurious* tuples - result includes

121 1803 66

222 1803 99

Normalization



Boyce Codd Normal Form, BCNF

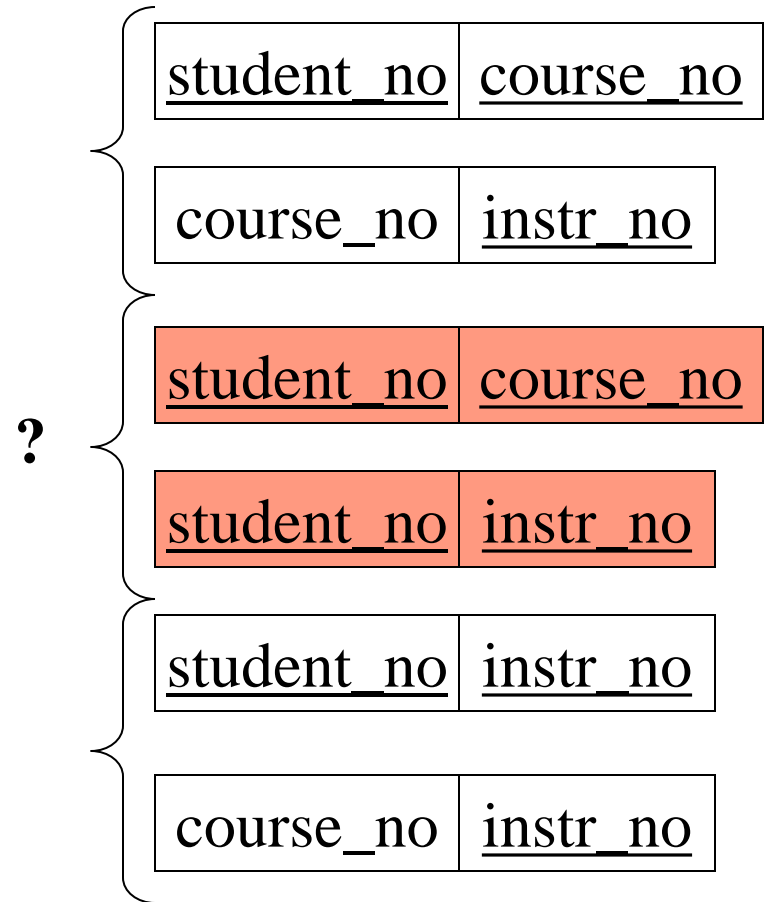
Which decomposition preserves all the information?

S#	C#
121	1803
121	1903
222	1803
222	1903

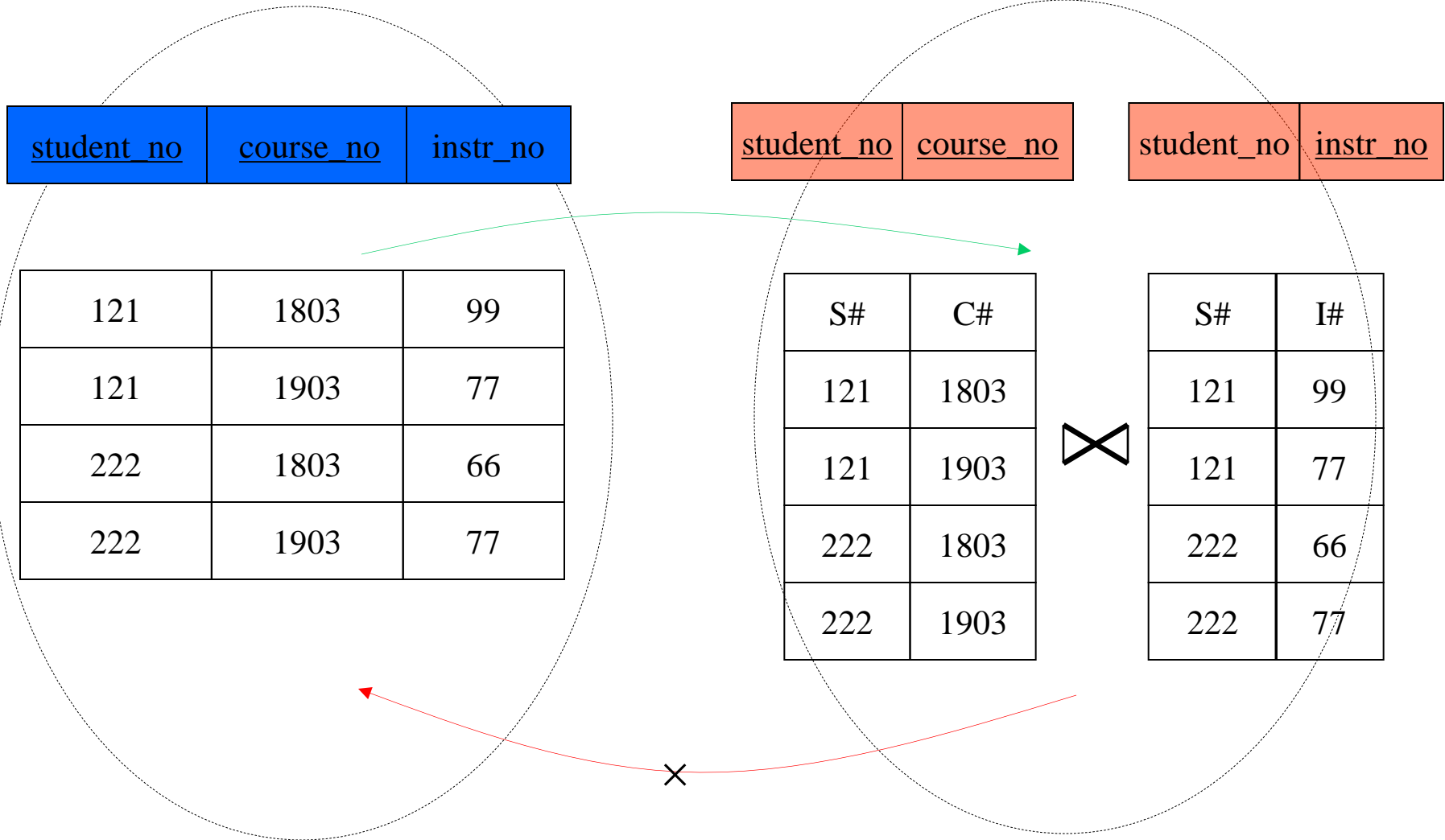
S#	I#
121	99
121	77
222	66
222	77

Joining these two tables leads to *spurious* tuples - result includes

121 1803 77
 121 1903 99
 222 1803 77
 222 1903 66



Normalization



Boyce Codd Normal Form, BCNF

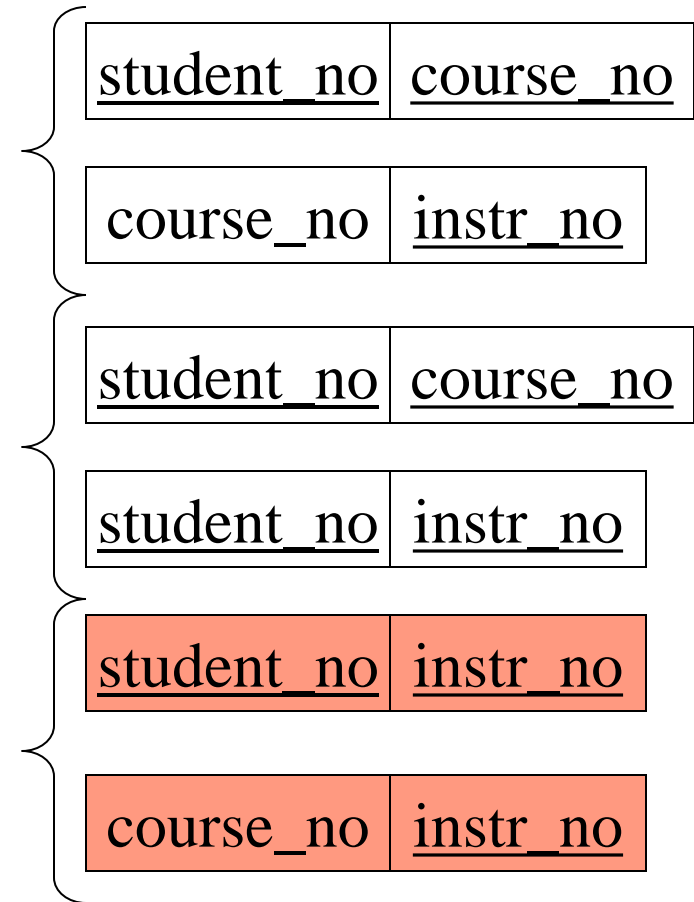
Which decomposition preserves all the information?

S#	I#
121	99
121	77
222	66
222	77

C#	I#
1803	99
1903	77
1803	66

Joining these two tables leads to **no spurious** tuples - result is:

121 1803 99
 121 1903 77
 222 1803 66
 222 1903 77



Boyce Codd Normal Form, BCNF

This decomposition preserves all the information.

S#	I#
121	99
121	77
222	66
222	77

C#	I#
1803	99
1903	77
1803	66

<u>student_no</u>	<u>instr_no</u>
-------------------	-----------------

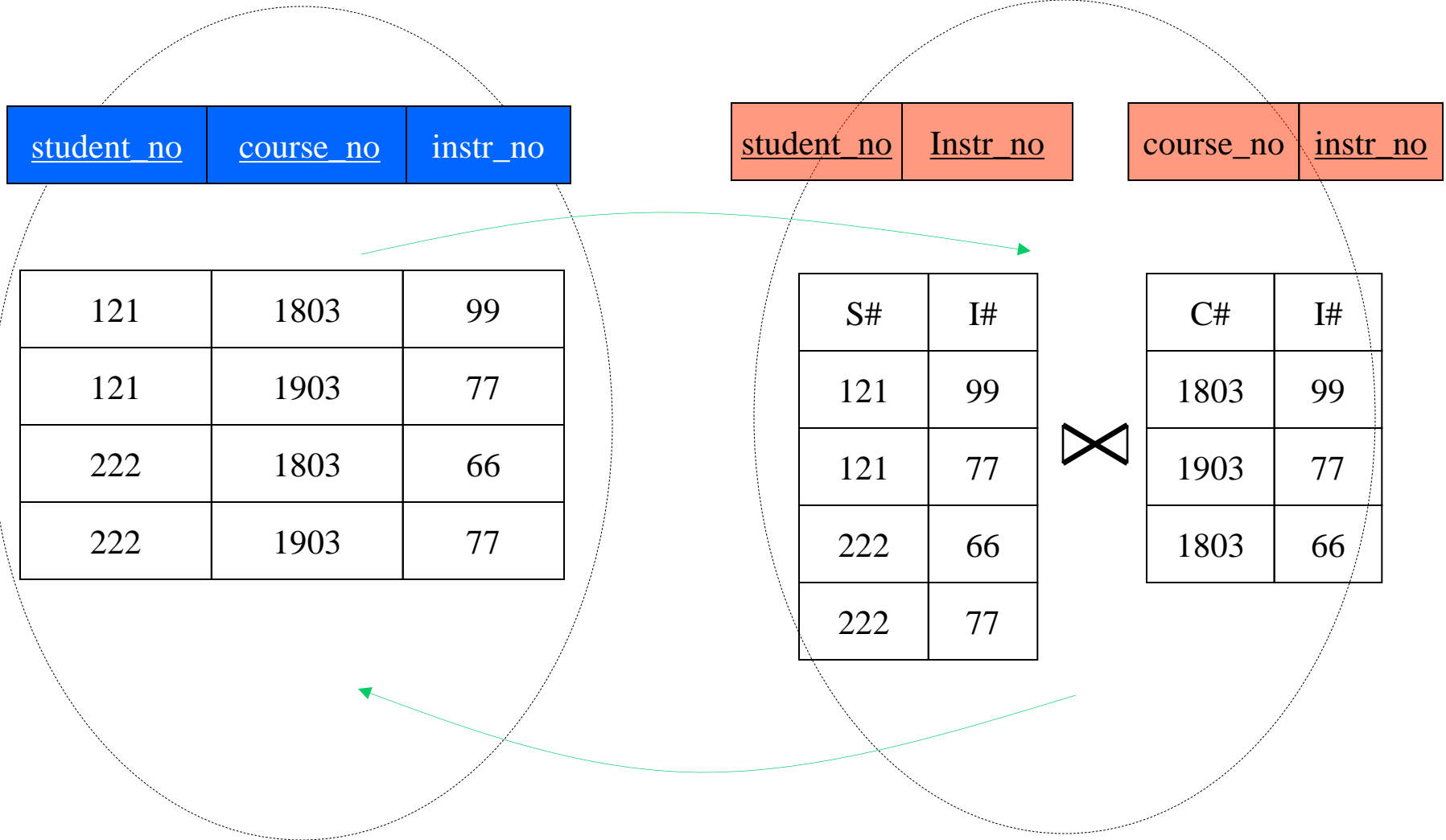
course_no	<u>instr_no</u>
-----------	-----------------

Only FD is $\text{instr_no} \longrightarrow \text{course_no}$

but the join preserves

$\{\text{student_no}, \text{course_no}\} \longrightarrow \text{instr_no}$

Normalization

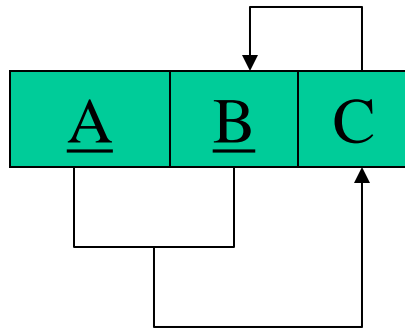


Boyce Codd Normal Form, BCNF

A relation schema is in BCNF if every determinant is a candidate key.

Boyce Codd Normal Form, BCNF

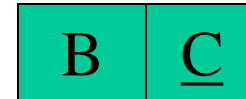
Given:



In 3NF

Not in BCNF

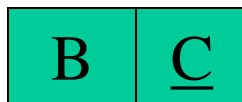
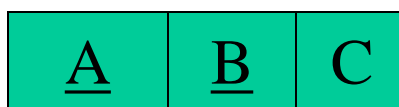
Lossless decomposition pattern:



In BCNF

But this could be where a database designer may decide to go

with:



- *Functional dependencies are preserved*
- *There is some redundancy*
- *Delete anomaly is avoided*

Outline: Lossless-join

- Basic definition of Lossless-join
- Examples
- Testing algorithm

- Basic definition of Lossless-join

A decomposition $D = \{R_1, R_2, \dots, R_m\}$ of R has the *lossless join property* with respect to the set of dependencies F on R if, for every relation r of R that satisfies F , the following holds,

$$*(\pi_{R_1}(r), \dots, \pi_{R_m}(r)) = r,$$

where $*$ is the natural join of all the relations in D .

The word loss in lossless refers to *loss of information*, not to loss of tuples.

- Example: decomposition-1

Emp_PROJ

<u>SSN</u>	<u>PNUM</u>	hours	ENAME	PNAME	PLOCATION
------------	-------------	-------	-------	-------	-----------

$F = \{SSN \rightarrow ENAME, PNUM \rightarrow \{PNAME, PLOCATION\}, \{SSN, PNUM\} \rightarrow hours\}$



R1

SSN	ENAME
-----	-------

R2

PNUM	PNAME	PLOCATION
------	-------	-----------

R3

SSN	PNUM	hours
-----	------	-------

Lossless join

- Example: decomposition-2

Emp_PROJ

SSN	PNUM	hours	ENAME	PNAME	PLOCATION
-----	------	-------	-------	-------	-----------

$F = \{SSN \rightarrow ENAME, PNUM \rightarrow \{PNAME, PLOCATION\},$
 $\{SSN, PNUM\} \rightarrow hours\}$



R1

ENAME	PLOCATION
-------	-----------

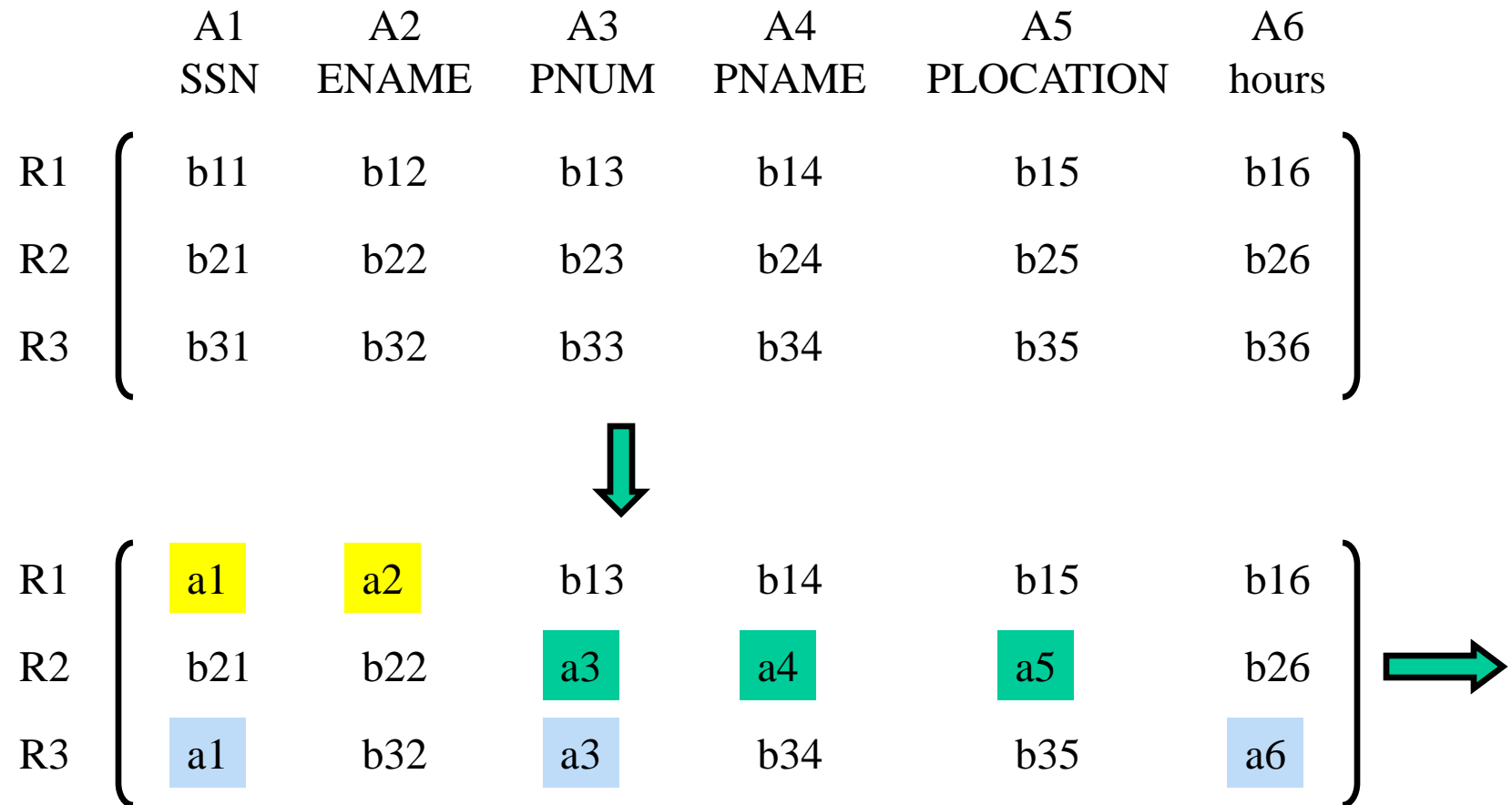
← Not lossless join

R2

SSN	PNUM	hours	PNAME	PLOCATION
-----	------	-------	-------	-----------

Normalization

- decomposition-1



Normalization



SSN \rightarrow ENAME

	SSN	ENAME				
R1	a1	a2	b13	b14	b15	b16
R2	b21	b22	a3	a4	a5	b26
R3	a1	a2	a3	b34	b35	a6



PNUM \rightarrow {PNAME, PLOCATION}

			PNUM	PNAME	PLOCATION	
R1	a1	a2	b13	b14	b15	b16
R2	b21	b22	a3	a4	a5	b26
R3	a1	a2	a3	a4	a5	a6

- decomposition-2

	A1	A2	A3	A4	A5	A6
	SSN	ENAME	PNUM	PNAME	PLOCATION	hours
R1	b11	b12	b13	b14	b15	b16
R2	b21	b22	b23	b24	b25	b26



R1	b11	a2	b13	b14	a5	b16
R2	a1	b22	a3	a4	a5	a6



SSN \rightarrow ENAME
 PNUM \rightarrow {PNAME, PLOCATION}
 {SSN, PNUM} \rightarrow hours

The matrix can not be changed!

Normalization

Why?

Decomposition-1: EMP_PROJ

a1	a2	b13	b14	b15	b16
b21	b22	a3	a4	a5	a6
a1	a2	a3	a4	a5	a6

R1

a1	a2
b21	b22

R2

b13	b14	b15
a3	a4	a5

R3

a1	b13	b16
b21	a3	b26
a1	a3	a6

R1

SSN	ENAME
-----	-------

R2

PNUM	PNAME	PLOCATION
------	-------	-----------

R3

SSN	PNUM	hours
-----	------	-------

Why?

Decomposition-1:

$$R1 * R3 = R13 =$$

a1	a2	b13	b16
a1	a2	a3	a6
b21	b22	a3	b26

$$R13 * R2 =$$

a1	a2	b13	b14	b15	b16
b21	b22	a3	a4	a5	a6
a1	a2	a3	a4	a5	a6

Normalization

Why?

Decomposition-2:
EMP_PROJ

b11	a2	b13	b14	a5	b16
a1	b22	a3	a4	a5	a6

R1

R1

ENAME	PLOCATION
-------	-----------

a2	a5
b22	a5

R2

SSN	PNUM	hours	PNAME	PLOCATION
-----	------	-------	-------	-----------

R2

b11	b13	b14	a5	b16
a1	a3	a4	a5	a6

Why?

Decomposition-2:

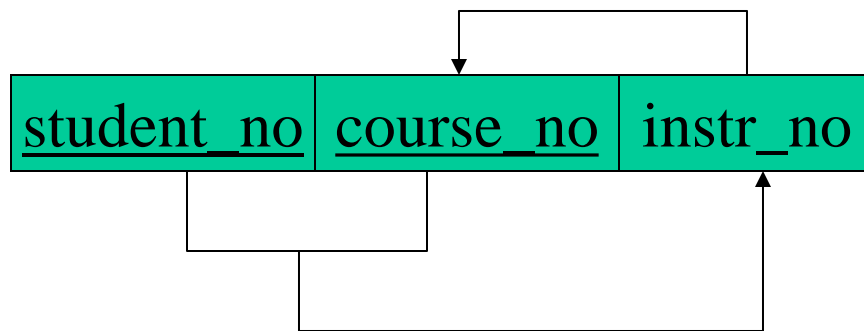
$R1 * R2 =$

b11	a2	b13	b14	a5	b16
a1	a2	a3	a4	a5	a6
b11	b22	b13	b14	a5	b16
a1	b22	a3	a4	a5	a6

Spurious tuples



Student-course-instructor:



*Instructor's teach one
course only*

*Student takes a course
and has one instructor*

$\{\text{student_no}, \text{course}\} \rightarrow \text{instr_no}$
 $\text{instr_no} \rightarrow \text{course_no}$

Normalization

<u>student_no</u>	<u>course_no</u>	instr_no
-------------------	------------------	----------



R1

Course_no	<u>instr_no</u>
-----------	-----------------

{student_no, course} → instr_no
instr_no → course_no

R2

<u>student_no</u>	<u>instr_no</u>
-------------------	-----------------

	A1	A2	A3		A1	A2	A3	
	stu-no	course-no	instr-no		stu-no	course-no	instr-no	
R1	b11	b12	b13	→	R1	a2	a3	
R2	b21	b22	b23		R2	a1	b22	a3

R1	b11	a2	a3
R2	a1	a2	a3

Normalization

<u>student_no</u>	<u>course_no</u>	instr_no
-------------------	------------------	----------



R1

Course_no	<u>instr_no</u>
-----------	-----------------

$\{ \text{student_no}, \text{course} \} \rightarrow \text{instr_no}$
 $\text{instr_no} \rightarrow \text{course_no}$

R2

<u>student_no</u>	<u>course_no</u>
-------------------	------------------

A1 A2 A3
 stu-no course-no instr-no

R1

b11	b12	b13
b21	b22	b23



A1 A2 A3
 stu-no course-no instr-no

R1

b11	a2	a3
a1	a2	b23

$\text{instr_no} \rightarrow \text{course_no}$



R1

b11	a2	a3
a1	a2	b23

Normalization

<u>student_no</u>	<u>course_no</u>	instr_no
-------------------	------------------	----------



R1

<u>student_no</u>	<u>instr_no</u>
-------------------	-----------------

{student_no, course} → instr_no
instr_no → course_no

R2

<u>student_no</u>	<u>course_no</u>
-------------------	------------------

	A1	A2	A3		A1	A2	A3
	stu-no	course-no	instr-no		stu-no	course-no	instr-no
R1	b11	b12	b13	→	a1	b12	a3
R2	b21	b22	b23		a1	a2	b23

R1	a1	b12	a3
R2	a1	a2	b23

Testing algorithm

input: A relation R , a decomposition $D = \{R_1, R_2, \dots, R_m\}$ of R , and a set F of function dependencies.

1. Create an initial matrix S with one row i for each relation R_i in D , and one column j for each attribute A_j in R .
2. Set $S(i, j) := b_{ij}$ for all matrix entries.
3. **For** each row i representing relation schema R_i **Do**
 {**for** each column j representing A_j **do**
 {**if** relation R_i includes attribute A_j **then**
 set $S(i, j) := a_j$;}
 }
4. Repeat the following loop until a complete loop execution results in no changes to S .

4. Repeat the following loop until a complete loop execution results in no changes to S .
 - { **for** each function dependency $X \rightarrow Y$ in F **do**
 - for** all rows in S which have the same symbols in the columns corresponding to attributes in X **do**
 - {make the symbols in each column that correspond to an attribute in Y be the same in all these rows as follows:
if any of the rows has an “a” symbol for the column, set the other rows to the same “a” symbol in the column. If no “a” symbol exists for the attribute in any of the rows, choose one of the “b” symbols that appear in one of the rows for the attribute and set the other rows to that same “b” symbol in the column;}}
5. If a row is made up entirely of “a” symbols, then the decomposition has the lossless join property; otherwise it does not.

Normalization

a1	a2	b13	b14	b15	b16
b21	b22	a3	a4	a5	b26
a1	b32	a3	b34	b35	a6

R1<SSN, ENAME>

a1	a2
b21	b22
a1	b32

R2<PNUM, PNAME, Plocation>

b13	b14	b15
a3	a4	a5
a3	b34	b35

R3<SSN, PNUM, hours>

a1	b13	b16
b21	a3	b26
a1	a3	a6

PNUM → {PNAME, PLOCATION}

<a3, a4, a5, a1, a3, a6>

<a3, b34, b35, a1, a3, a6>